

# Hyperelastic Image Registration With an Application to PET Reconstruction

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Image registration is one of the challenging problems in image processing. Given are two images that are taken for example at different times, from different devices or perspectives. The goal is to determine a reasonable transformation, such that a transformed version of one of the images is similar to the second one.

In this talk, we give a brief introduction to this fascinating problem and present typical areas of applications. We outline a state-of-the-art mathematical model, that is based on a flexible variational setting. We discuss important features such as appropriate data fitting, regularization, and the integration of additional constraints.

A focus of the talk is on hyperelastic image registration, which is motivated by an application from positron emission tomography (PET) cardiac imaging. More specifically, we present a hyperelastic regularizer and we show that this regularizer enables the recovery of large

and highly non-linear transformations. We also show that this regularization results in diffeomorphic mappings. The price to be paid is a non-convex but polyconvex objective function.

We also present a stable and efficient numerical implementation of hyperelastic registration. This implementation is based on the discretize then optimize paradigm and uses a sophisticated computation of the discrete analogues of the three invariants of the transformation tensor: lengths, areas and volumes. We show several numerical examples that illustrate the potential of the hyperelastic regularizer. We also show the mass-preserving registration of cardiac PET images, where hyperelastic regularization is mandatory.